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## COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE- A Study of Some Attitude and Control Options Compatible with the Performance of Earth-Pointing Experiments by the AAP Cluster

TM- 70-1022-2

DATE- February 3, 1970

FILING CASE NO(S)- 620

AUTHOR(S)- J. J. Fearnsides

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(NASA-CR-112394) A STUDY OF SOME ATTITUDE AND CONTROL OPTIONS COMPATIBLE WITH THE PERFORMANCE OF EARTHPOINTING EXPERIMENTS BY THE AAP CLUSTER (Bellcomm, Inc.) 23 p

### ABSTRACT

This paper is concerned with the impact on the Saturn V Workshop attitude control systems of acquiring and holding an Earth-pointing attitude. Three different methods of acquiring this attitude from the solar inertial attitude are discussed. The primary practical results are:

1. The CMGs have the capability to stabilize the Workshop in the XIOP/Z-LV mode (X-axis in the orbital plane, Z-axis along the local vertical).

2. The maneuvers required to acquire this Earth-pointing attitude can be performed by the CMGs only if the RCS[1] is used to desaturate (or dump) the bias momentum which accumulates in the CMGs.

These results (which are based on the variation of the resultant CMG angular momentum vector) lead to the conclusion that the CMGs could be the primary control system in XIOP/Z-LV with the RCS used only for dumping. This control option can perform Earth-pointing attitude control of the Workshop for less than 20% of the propellant which would be required if attitude control were performed by RCS alone.

There is, in addition, the important technical result that an accumulation of bias momentum in the CMGs as a result of a constant external torque can occur only about the spin axis (in this case, the axis which is perpendicular to the orbital plane). It is shown that a constant torque applied about a spacecraft axis which is in the plane perpendicular to the spin axis

[1]RCS denotes here a generic reaction control system which on the AAP Cluster can be either the TACS on the SWS or the RCS on the CSM.

will produce a periodic momentum change in the CMGs. This result is especially important for the example which is considered in this paper since the important disturbance torques acting on an Earth-pointing satellite are constant relative to the spacecraft.

Finally, it is shown that the momentum which does accumulate in the CMGs due to the torque about the spin axis can be dumped either with an RCS or by performing vehicle maneuvers on the dark side of the orbit.

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SUBJECT: A Study of Some Attitude and Control Options Compatible with the Performance of Earth-Pointing Experiments by the AAP Cluster

DATE: February 3, 1970

FROM: J. J. Fearnsides

TM-70-1022-2

TECHNICAL MEMORANDUM

# I. INTRODUCTION

This paper is concerned with the impact on the Saturn V Workshop (SVWS) attitude control systems of acquiring and holding an Earth-pointing attitude. Three different methods of acquiring this attitude from the solar inertial (SI) attitude are discussed. These attitude (or acquisition) options are:

A01. Noon acquisition of orbital rate,

A02. Midnight acquisition of orbital rate with preceding roll maneuver, and

A03. Earth-pointing acquisition immediately preceding experiment zone.

In addition, four different control options (CO) are considered. They are:

C01. Control moment gyros (CMGs) for acquisition, stabilization and momentum dumping.

C02. Reaction control system (RCS)[1] for acquisition and stabilization.

C03. CMGs for acquisition and stabilization; RCS for momentum dumping.

C04. CMGs for stabilization; RCS for acquisition and momentum dumping.

The primary practical results are the following:

1. If the CMGs have the capability to stabilize a given inertial attitude, then, with proper initialization, they have the capability to stabilize the corresponding

[1]In this paper RCS will be used to denote both the TACS and the reaction control system of the CSM.

Earth-pointing attitude.[1]

2. The maneuvers required to acquire an Earth-pointing attitude can be performed by the CMGs only if some assistance is provided by the RCS for dumping the bias momentum accumulated by the CMGs.

These results are based on an analysis of the variation of the components of the total CMG angular momentum vector, not on the details of gimbal angle motion.

There is, in addition, the important technical result that an accumulation of bias momentum in the CMGs as a result of a constant external torque can occur only about an axis perpendicular to the orbital plane. It is shown that a constant torque applied about a spacecraft axis which lies in the orbital plane produces an oscillating momentum change. Some bias accumulation is possible due to the diurnal bulge effect in the atmosphere but this is shown to be quite small.

The organization of this material is as follows: In Section II a detailed analysis of the capability of the CMGs to stabilize an Earth-pointing attitude is presented; Section III contains an evaluation of the various attitude and control options; and Section IV is reserved for some concluding comments.

## II. ATTITUDE HOLD OF AN EARTH-POINTING ORIENTATION WITH CMGS

The equations of motion relative to a set of coordinates which are fixed in a spacecraft containing CMGs is given by

$$I\dot{\underline{\omega}} + \tilde{\omega}I\underline{\omega} + \dot{\underline{H}} + \tilde{\omega}\underline{H} = \underline{N} \tag{1}$$

where: $I$ = the $3 \times 3$ inertia tensor,

$\underline{\omega}$ = a $3 \times 1$ matrix representing the angular velocity of the spacecraft relative to a set of inertial coordinates,

$\tilde{\omega}$ = the $3 \times 3$ skew-symmetric matrix which is isomorphic to the vector cross-product operator,

[1]An Earth-pointing coordinate system is defined to correspond to an inertial coordinate system if they differ only by an angular displacement about an axis which is perpendicular to the orbital plane. This displacement is given by $\omega_o t$ where $\omega_o$ equals the orbital rate and t is the variable representing time.

$\underline{N}$ = a 3 × 1 matrix representing the total external torque vector acting on the spacecraft, and

$\underline{H}$ = a 3 × 1 matrix which represents the vector sum of the individual spin angular momenta contributed by each CMG. That is, for a system containing n CMGs with the spin angular momentum of the $i^{th}$ CMG given by $\underline{h}_i$,

(2) $$\underline{H} = \sum_{i=1}^{n} \underline{h}_i \; .$$

The dots over $\underline{\omega}$ and $\underline{H}$ in (1) denote the time rate of change of those vectors as measured in a set of coordinates fixed in the spacecraft.

For the purpose of investigating the attitude-hold momentum requirements on the CMGs of an Earth-pointing spacecraft in a circular orbit, let

(3) $$\underline{\omega} = \underline{\omega}_o + \underline{\Delta\omega}$$

where $\underline{\omega}_o$ is the constant orbital rate about the axis which is perpendicular to the orbital plane (and which will be called the POP axis). Substitution of (3) into (1) yields

(4) $$I\underline{\Delta\dot{\omega}} + (\tilde{\omega}_o + \Delta\tilde{\omega})I(\underline{\omega}_o + \underline{\Delta\omega}) + \underline{\dot{H}} + (\tilde{\omega}_o + \Delta\tilde{\omega})\underline{H} = \underline{N} \; .$$

The assumption is now made that the CMG control system is capable of maintaining the actual state of the system identically equal to the desired state. Then, $\underline{\Delta\omega} = \underline{0}$ and (4) becomes

(5) $$\underline{\dot{H}} + \tilde{\omega}_o\underline{H} = \underline{N} - \tilde{\omega}_o I \underline{\omega}_o \overset{\Delta}{=} \underline{T} \; .$$

This assumption is certainly justified for the determination of the CMG momentum capacity which will be required to stabilize a particular attitude. The neglected terms are merely perturbations [1] from the solution of (5).

Since $\underline{\omega}_o$ is constant, (5) is a linear, time-invariant, vector differential equation which can be solved (e.g. by Laplace transform) once the external torques are defined. Before this is done, however, two coordinate systems will be defined.

The orbit-referenced coordinate system (ORS) is specified by axes $X_o$, $Y_o$, and $Z_o$ which are defined as follows:

$Y_o$ - perpendicular to the orbital plane (POP); positively directed to make an acute angle with the ecliptic North Pole,

$Z_o$ - along the local vertical; positively directed towards the geocenter,

$X_o$ - completes the orthogonal right triad.

The geometric coordinate system (GS) is defined in Figure 1. Note that when the GS is aligned with the ORS the spacecraft is in the XIOP/Z-LV[1] attitude mode.

Equation (5) will be solved in the ORS. For the case where the GS is aligned with the ORS, the solutions will yield the required CMG momentum variation relative to spacecraft coordinates. When these systems are not aligned, e.g. if they differ by a constant X-axis transformation, the solution of (5) must be transformed into the GS.

Relative to the ORS then, the transpose of $\underline{\omega}_o$ is given by $\underline{\omega}_o' = [0 \quad \omega_o \quad 0]$ and the expanded and transformed version of (5) for arbitrary $\underline{T}(t)$ is given by

$$
\begin{aligned}
sH_x(s) + \omega_o H_z(s) &= T_x(s) + H_x(0+) \\
sH_y(s) &= T_y(s) + H_y(0+) \qquad (6) \\
-\omega_o H_x(s) + sH_z(s) &= T_z(s) + H_z(0+)
\end{aligned}
$$

---

[1] X-axis in the orbital plane, Z-axis along local vertical.

where t is the time variable and the initial time, $t_o$, is taken equal to zero. Notice that the Y-axis equation exhibits the behavior that is expected when the CMGs hold an inertial attitude. That is, the momentum change about a given axis is equal to the integral over time of the torque which is acting about that axis. This is because the Y-axis is the axis of rotation and is fixed relative to a set of inertial coordinates. On the other hand, the equations for the momentum variation about the axes which lie in the orbital plane are coupled. The simultaneous solution of these equations leads to the characteristic equation $s^2 + \omega_o^2 = 0$ which implies that the natural response of this system is an undamped oscillation of angular frequency $\omega_o$.

These general principles are now applied to the problem of holding the SVWS in XIOP/Z-LV and acted upon by gravity-gradient torques.[1] The problem is made non-trivial by assuming a non-diagonal inertia tensor for the Workshop. That is, the geometric axes will be kept XIOP/Z-LV instead of the principal axes. The gravity-gradient torque for this problem is constant under the assumptions made above as are the effects of the $\tilde{\omega}_o I \underline{\omega}_o$ term. A time domain solution of (5) under these circumstances is given by

$$H_x(t) = \{-T_z/\omega_o + [H_x(0+) + T_z/\omega_o] \cos \omega_o t$$

$$- [H_z(0+) - T_x/\omega_o] \sin \omega_o t\}$$

$$(7) \qquad H_y(t) = H_y(0+) + \int_0^t T_y dt$$

$$H_z(t) = \{T_x/\omega_o + [H_z(0+) - T_x/\omega_o] \cos \omega_o t$$

$$+ [H_x(0+) + T_z/\omega_o] \sin \omega_o t\}$$

---

[1]Since the CMGs can hold the spacecraft's attitude very close to the nominal XIOP/Z-LV, the end of the cylinder is pointed into the "wind" as are the edges of the solar panels. Using the mass properties given in [2], the bias momentum accumulation in the CMGs due to aerodynamic effects is conservatively estimated to be 35 ft-lb-sec/orbit.

where $T_x = -4\omega_o^2 I_{yz}$,

$T_y = 3\omega_o^2 I_{xz}$,

$T_z = \omega_o^2 I_{xy}$, and

$I_{ij}$, $i,j = x,y,z$ are the appropriate elements of the inertia tensor.

Note that the oscillatory terms in the equations for $H_x$ and $H_z$ can be eliminated by a proper choice of $H_x(0+)$ and $H_z(0+)$.[1,2] Further, note that the initial time $t_o$ could have been chosen arbitrarily, that is the 0+ reference in this problem is not related to any orbital time. Therefore, for example, the time of firing of a set of thrusters which produce an almost impulsive torque would be an appropriate choice for $t_o$.

Some representative numbers for equation (7) as it represents the SVWS in the XIOP/Z-LV orientation and in a 235 nautical mile circular orbit are [2]

$$T_x/\omega_o \simeq 86 \text{ ft-lb-sec}$$

$$T_z/\omega_o \simeq 1.5 \text{ ft-lb-sec}$$

$$\int_0^{2\pi/\omega_o} T_y \, dt \simeq 4{,}645 \text{ ft-lb-sec}$$

Unfortunately, the effect of an unusually large $I_{xz}$ term appears about the axis of rotation which means that 4,645 ft-lb-sec of momentum must be dumped per orbit. It is shown in Appendix A that this dumping can be achieved by the gravity-gradient torque if a single-axis maneuver is permitted on the dark side of the orbit. For the AAP Earth-pointing modes, however, there are good arguments for using RCS thrust for dumping (See Table 1 in §2).

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[1]This is not used, however, in the example presented in this paper.

[2]Also, note that, since the position of $\underline{H}$ relative to spacecraft coordinates can be varied to produce a specified torque, the solution of (5) when $\underline{T} = \underline{0}$ must be $\underline{H} = \underline{0}$ to avoid needless gimbal motion.

An alternate method of reducing the large, Y-axis bias momentum accumulation is to hold an attitude which differs only by a small, Y-axis rotation, $\theta$, from XIOP/Z-LV. The gravity-gradient torque for this condition is given by

$$\underline{N}_{gg} = 1.5\omega_o^2 \begin{bmatrix} I_{xy} \sin 2\theta - I_{yz}(1+\cos 2\theta) \\ (I_{zz}-I_{xx}) \sin 2\theta + 2I_{xz} \cos 2\theta \\ I_{xy}(1-\cos 2\theta) - I_{yz} \sin 2\theta \end{bmatrix} \tag{8}$$

The Y component of this expression is nulled when $\theta = -4.4$ degrees for the SVWS. This is the angle which removes the X-axis component (in the ORS) of the vector $I\underline{a}_{zo}$[1] where $\underline{a}_{zo}$ is a unit vector along the local vertical (the $-Z_o$ axis in the ORS). No significant increase in the momentum requirement about the X and Z axes of the GS is caused by this rotation.

## III. AAP APPLICATIONS

The establishment of the mathematical model for the momentum requirements of the CMGs of the SVWS for holding a local vertical attitude enables an evaluation of various possible options for performing Earth resources experiments. As mentioned in §I, three attitude options and four control options were studied. The results are presented in matrix form in Table 1 and presented in more detail in the sequel.

The first attitude option (A01), the noon acquisition of the local vertical attitude, is considered because it is easiest to perform from a control system standpoint. If it were found that the thermal and power systems could withstand two or more consecutive orbits of the XIOP/Z-LV attitude, then this option would be worthy of consideration. The principal results are:

1. A01 could be accomplished by three CMGs without any RCS assist (C01). This includes the gravity-gradient dump maneuver described in Appendix A which is performed on the dark side of the orbit. This is illustrated in Figure 2. The peak-to-peak (PTP) variation in $|\underline{H}|$ is 7550 ft-lb-sec when only the gravity-gradient

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[1] Recall that, using this notation, the gravity-gradient torque is given by $\underline{N}_{gg} = 3\omega_o^2 \tilde{a}_{zo} I \underline{a}_{zo}$.

## TABLE 1

### A MATRIX OF ATTITUDE AND CONTROL OPTIONS FOR PERFORMING EARTH-POINTING EXPERIMENTS

| | Control Option 1 CMGs Alone | Control Option 2 RCS Alone [1] | Control Option 3 CMG/RCS Dump | Control Option 4 CMG/RCS Dump, Acq. |
|---|---|---|---|---|
| Attitude Option 1 (Noon) | Peak-to-peak $\Delta\lvert\underline{H}\rvert = 7550$ ft-lb-sec Includes gravity gradient dump. | 1,000 lb-sec impulse required per orbit for non-consecutive orbits. | 200 lb-sec impulse per orbit for dumping. Peak-to-peak $\Delta\lvert\underline{H}\rvert \approx 4{,}800$ ft-lb-sec. | 800 lb-sec impulse required per orbit for non-consecutive orbits. |
| Attitude Option 2 (Midnight) | Exceeds CMG capability unless 4.4 deg. Y-axis offset is controlled. | 1,160 lb-sec impulse required per orbit for non-consecutive orbits. | 185 lb-sec impulse per orbit could reduce peak-to-peak $\Delta\lvert\underline{H}\rvert$ to $\approx 4{,}800$ ft-lb-sec. | 960 lb-sec impulse required per orbit for non-consecutive orbits. |
| Attitude Option 3 (Over target) | ±8,000 ft-lb-sec maneuvering requirement exceeds CMG capability. | 1,310 lb-sec impulse for 60 deg. exp. pass. Approx. 2,000 lb-sec for 120 deg. exp. pass. | Not applicable | Not applicable |

[1] All RCS impulse estimates are based on a 32.5 foot moment arm about the Y and Z axes.

torque is considered. Hence, this control option would be impossible if one of the CMGs were disabled.

2. The propellant consumption estimate for A01 if RCS control is used exclusively (C02) is approximately 1,000 lb-sec/orbit for one orbit. This includes an extra 80 lb-sec to account for the effects of the aerodynamic torque and other contingencies.

3. If, instead of the gravity-gradient dump maneuver in C01, the dumping is done by the RCS (C03), then the rest of A01 could be performed with 2 CMGs. The propellant requirement estimate for C03 is 200 lb-sec orbit. This includes an estimate for initializing the momentum vector such that the total change in $|H|$ is minimized. If this is done the PTP change in $|\underline{H}|$ is approximately 4,800 ft-lb-sec.

4. Since most of the RCS propellant in C02 is required for maneuvering, C04 is relatively unattractive. About 200 lb-sec/orbit of propellant could be saved by holding attitude with CMGs but 800# sec/orbit would still be required.

Attitude option 2 (A02) is illustrated in Figure 3. The local vertical attitude is acquired sequentially. First, a roll maneuver is initiated at the sunset terminator to align the experiment window with the local vertical at midnight where the roll rate is removed and the orbital rate is initiated about the pitch (Y) axis. Then, the spacecraft stays in a local vertical attitude for one full orbit or until the midnight following acquisition at which time the orbital rate is removed and the return roll maneuver rate initiated. Finally, the roll rate is removed at the sunrise terminator. This attitude requires more control effort but allows a longer experiment interval for the same duration in the local vertical attitude. The results obtained by considering the cost of A02 with each of the control options are as follows:

1. Figure 4 illustrates the variation of the x, y and z components of $\underline{H}$ measured in spacecraft coordinates over the solar inertial attitude and through all the steps of A02. Since the dark side roll maneuver does not include a gravity-gradient dump maneuver, the Y axis accumulates momentum because of the bias gravity-gradient torque mentioned in §II. This produces a

large transient effect[1] in the Y and Z axes during the return roll maneuver. The combination of these two effects results in a momentum requirement that exceeds the capacity of the CMGs (±6,000 ft-lb-sec). Thus, without any RCS assist, this combination of options is not considered feasible unless the nominal attitude includes the 4.4 degree offset mentioned in §II.

2. If the same 80 lb-sec estimate for aerodynamic torque and other contingencies is included for the A02-C02 combination as it was for A01-C02, then the propellant consumption is 1,160 lb-sec/orbit for one orbit.

3. Figure 5 illustrates the variation of the x, y and z components of $\underline{H}$ relative to spacecraft coordinates if 3,891 ft-lb-sec of Y-axis momentum is dumped immediately after acquisition of the orbital rate. This profile costs 120 lb-sec of propellant/orbit. Another 65 lb-sec of propellant could, if judiciously applied, reduce the variation of the x and z components of $\underline{H}$ to an insignificant level.

4. As in the A01-C04 combination, A02-C04 saves 200 lb-sec relative to their corresponding control option, i.e. C02. Since the CMGs are capable of performing A01 and A02 with less RCS assist it is inefficient to use them only for attitude hold.

The third attitude option (A03) is illustrated in Figure 6. This is the best option for the thermal and power systems but has the most severe impact on the attitude control systems. The numbers given below for the various control options associated with A03 correspond to a 60 degree experiment interval. If this interval is increased to 120 degrees, the impact on the controller[2] increases by a factor of 1.6 and the roll maneuver must be initiated at the terminator instead of orbital 6 a.m. Further, since the time in local vertical is relatively small, only control options C01 and C02 are considered.

If 80 lb-sec is added for contingencies, the propellant consumption estimate for A03-C02 is 1310 lb-sec/orbit.

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[1] The principles which were outlined in §II can be generalized to provide a mathematical model for the roll maneuver since it involves spinning at a constant rate about a spacecraft axis. Since the X axis is now the spin axis, there is coupling between the Y and Z axis motions.

[2] That is, the fuel consumption of the RCS or the momentum variation of the CMGs.

This is only 150 lb-sec more than the cost of A02-C02. Therefore, if a 60 degree experiment interval is satisfactory, the requirements imposed by the thermal and power systems might dictate the choice of A03-C02 rather than A02-C02.

Finally, it is easy to show that A03 cannot be performed using only CMG control (i.e. C01). The primary problem is that, even though the required rate for the maneuver from ② to ③ in Figure 6 is only the negative of the orbital rate (.062 deg/sec), the required change in momentum to remove this rate and initiate the orbital rate, $\omega_o$, for local vertical acquisition (a rate change equal to $2\omega_o$) is approximately 8,000 ft-lb-sec. Since the reverse maneuver must be performed at ④ in Figure 6 (i.e. a $\Delta H_y = -8,000$ ft-lb-sec), the total required change is seen to exceed the ±6,000 ft-lb-sec capacity of the SVWS CMGs.

## IV. SUMMARY AND CONCLUSIONS

It is concluded from the results summarized in Table 1 that the CMGs may be used to good advantage to minimize the amount of RCS propellant required to perform the Earth-pointing experiments. In fact, the CMGs could be the primary system which requires only an occasional assist from the RCS.

Another point which should be made here is that the momentum profile illustrated in Figure 5 can be achieved in such a way that the gimbals of the CMGs remain away from their stops. Further, this momentum profile can be achieved with either two or three CMGs as long as some form of momentum dumping strategy is available. The question of whether or not the control law which is selected for controlling the SVWS in the solar inertial mode will also be acceptable for control in an Earth-pointing mode is presently being studied.

There is another reason for preferring C03 to C02 with either A01 or A02. That is, the CMGs can control the spacecraft attitude much more accurately than an RCS whose control law requires (at present) a 0.5 degree deadband. This means, in turn, that the Earth Resources Experiments can be pointed with better accuracy.

Finally, some important technical results are presented in §2. Chief among these is the demonstration of the fact that bias momentum accumulation is primarily about the axis which is perpendicular to the orbit plane. A detailed study of §2 will also reveal a fundamental difference in the way CMGs oppose external torques acting on the vehicle if the desired attitude is spinning relative to a set of inertial coordinates.

ACKNOWLEDGEMENT

The excellent assistance in programming and computation provided by Mrs. P. R. Dowling is appreciatively acknowledged.

J. J. Fearnsides

1022-JJF-cf J. J. Fearnsides

Attachment
Appendix A

## REFERENCES

1. Kranton, J., "Application of Optimal Control Theory to Attitude Control with Control Moment Gyros," D.Sc. Dissertation, George Washington University, February, 1970.

2. Hough, W. W., "AAP Cluster Mass Properties and CMG Control Capability," Memorandum for File, B69-11054, Bellcomm, Inc., Washington, D.C., November 20, 1969.

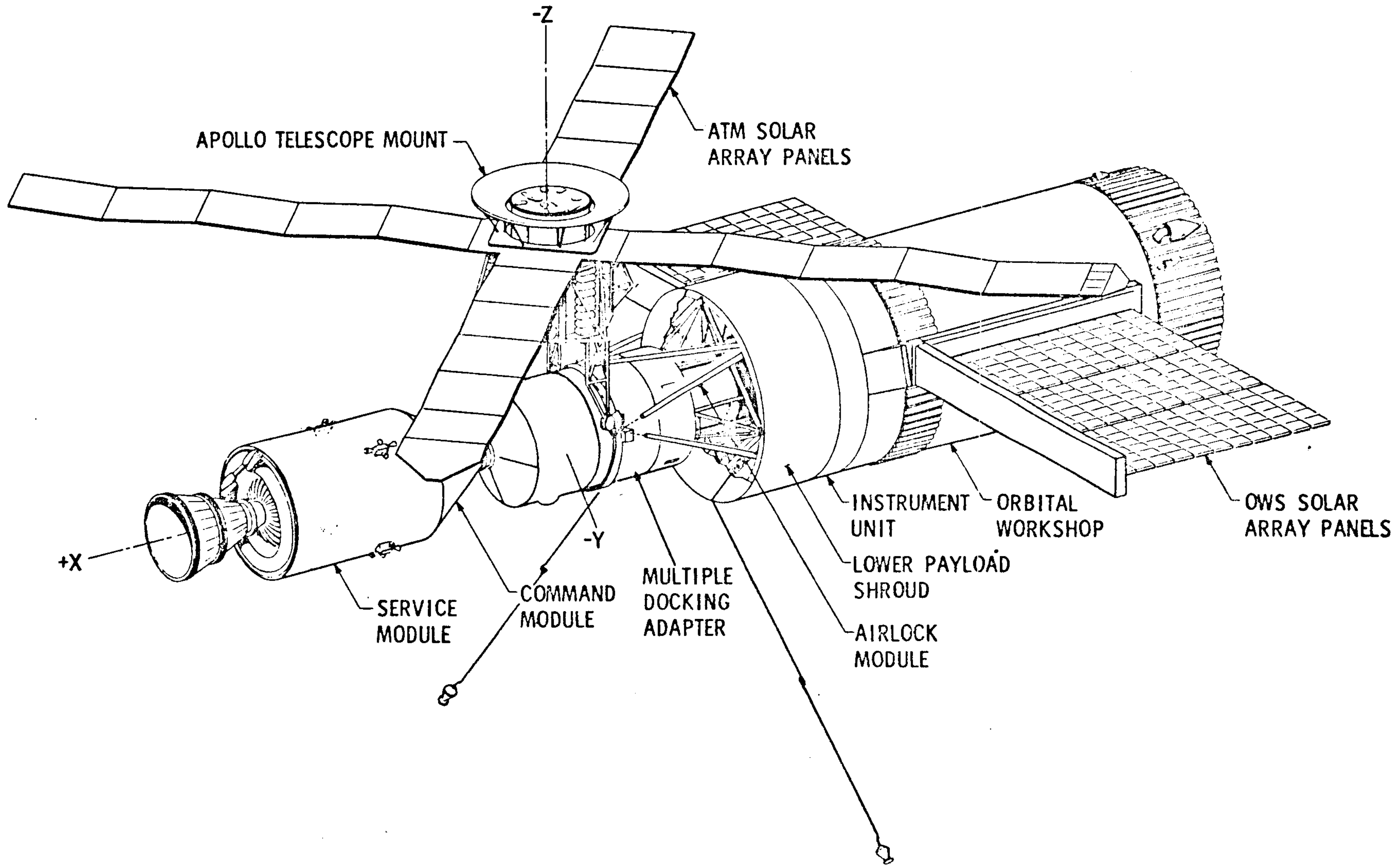
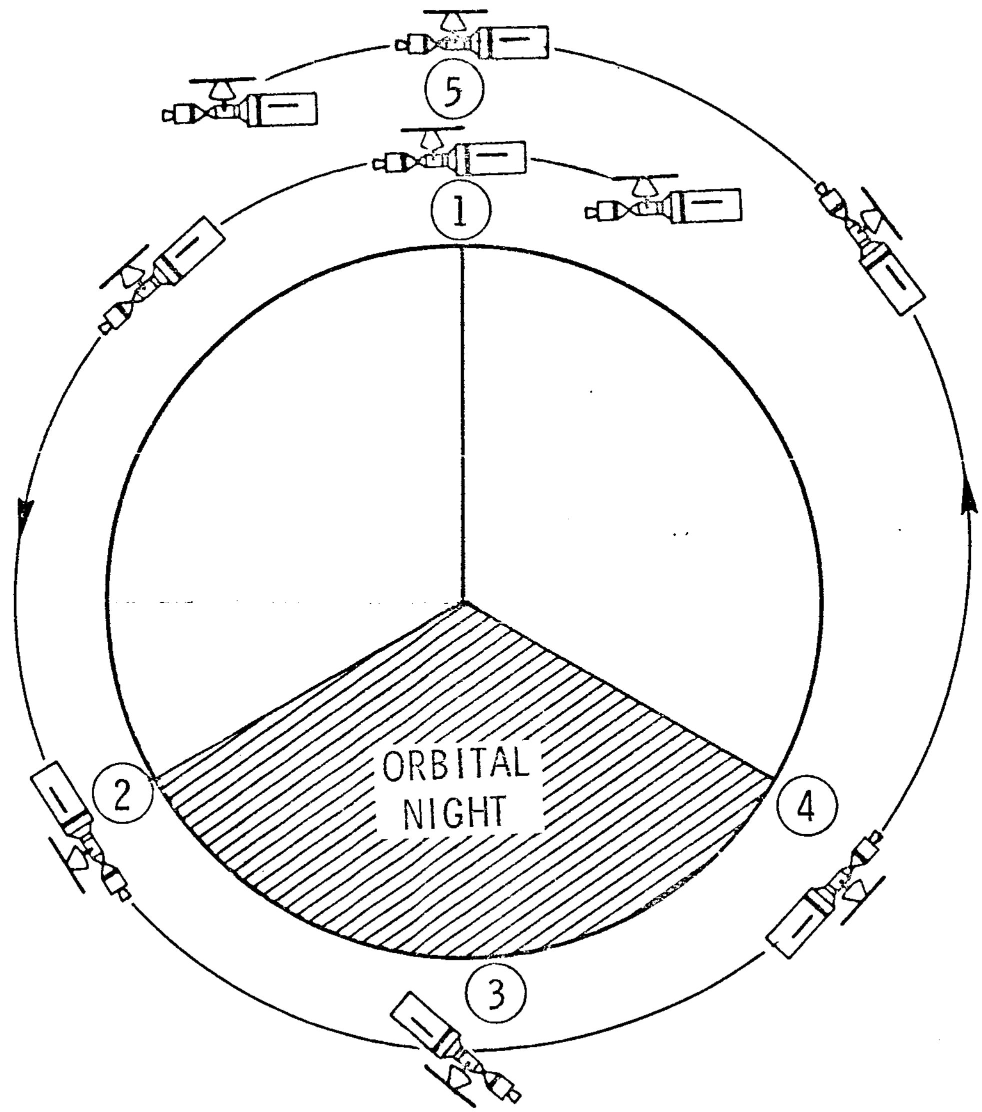



FIGURE 1 - SATURN V WORKSHOP

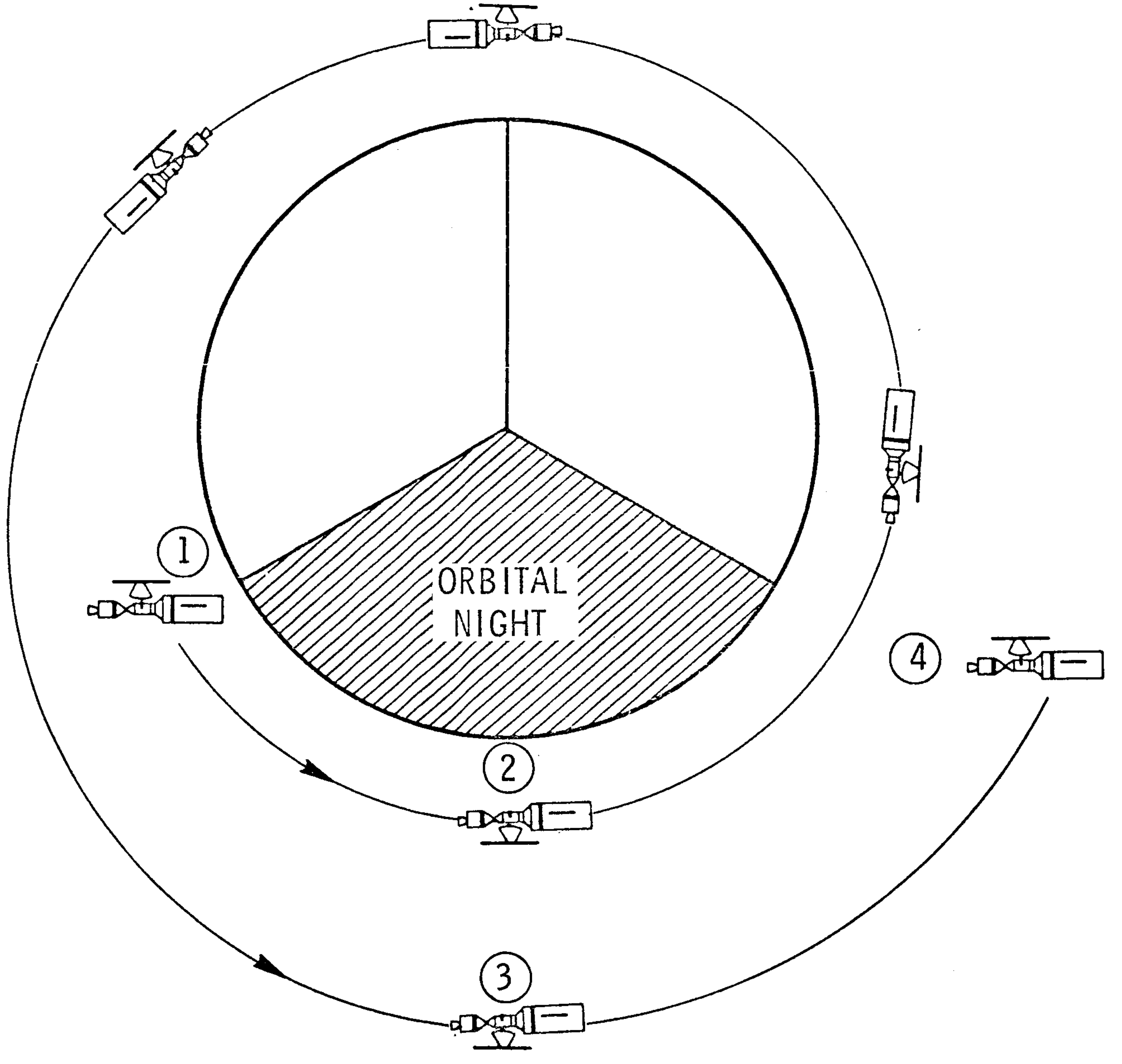
ATTITUDE OPTION 1




1. ACQUIRE LOCAL VERTICAL FROM SOLAR INERTIAL.
2. INITIATE GRAVITY-GRADIENT DUMP MANEUVER.
3. REVERSE PITCH RATE AND CONTINUE DUMPING.
4. REACQUIRE LOCAL VERTICAL BY REMOVING PITCH RATE.
5. REACQUIRE SOLAR INERTIAL.

FIGURE 2

ATTITUDE OPTION 2




(1) INITIATE ROLL MANEUVER TO ALIGN EXP. AXIS WITH THE LOCAL VERTICAL AT ORBIT MIDNIGHT.

(2) REMOVE ROLL RATE, INITIATE PITCH ORBITAL RATE.

(3) REMOVE PITCH ORBITAL RATE, INITIATE RETURN ROLL RATE.

(4) REMOVE ROLL RATE, SOLAR INERTIAL REACQUIRED.

FIGURE 3

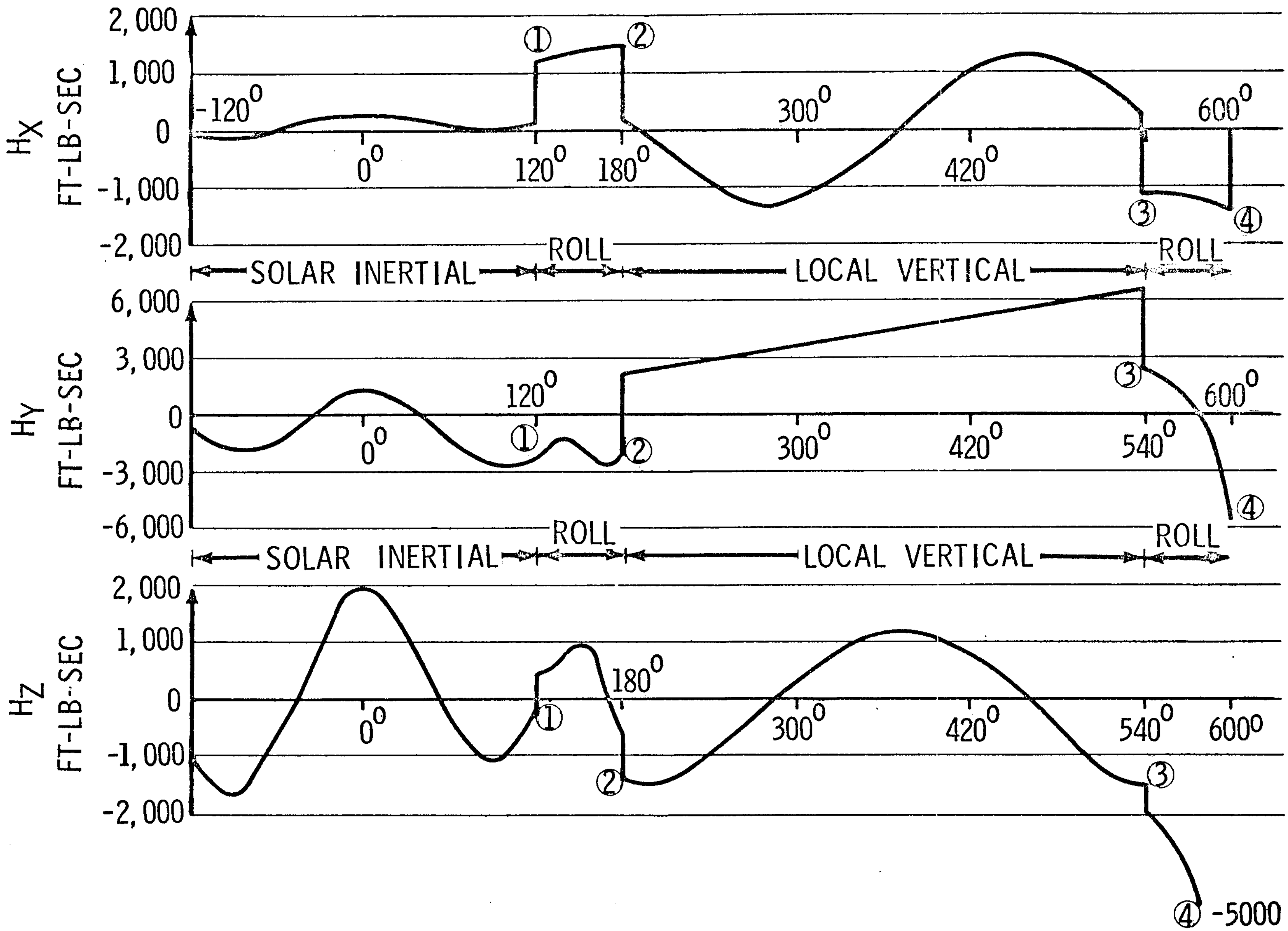



FIGURE 4 - ATTITUDE OPTION 2 - CONTROL OPTION 1
CMG MOMENTUM VARIATION VS. ORBIT ANGLE ($\beta$=45°)

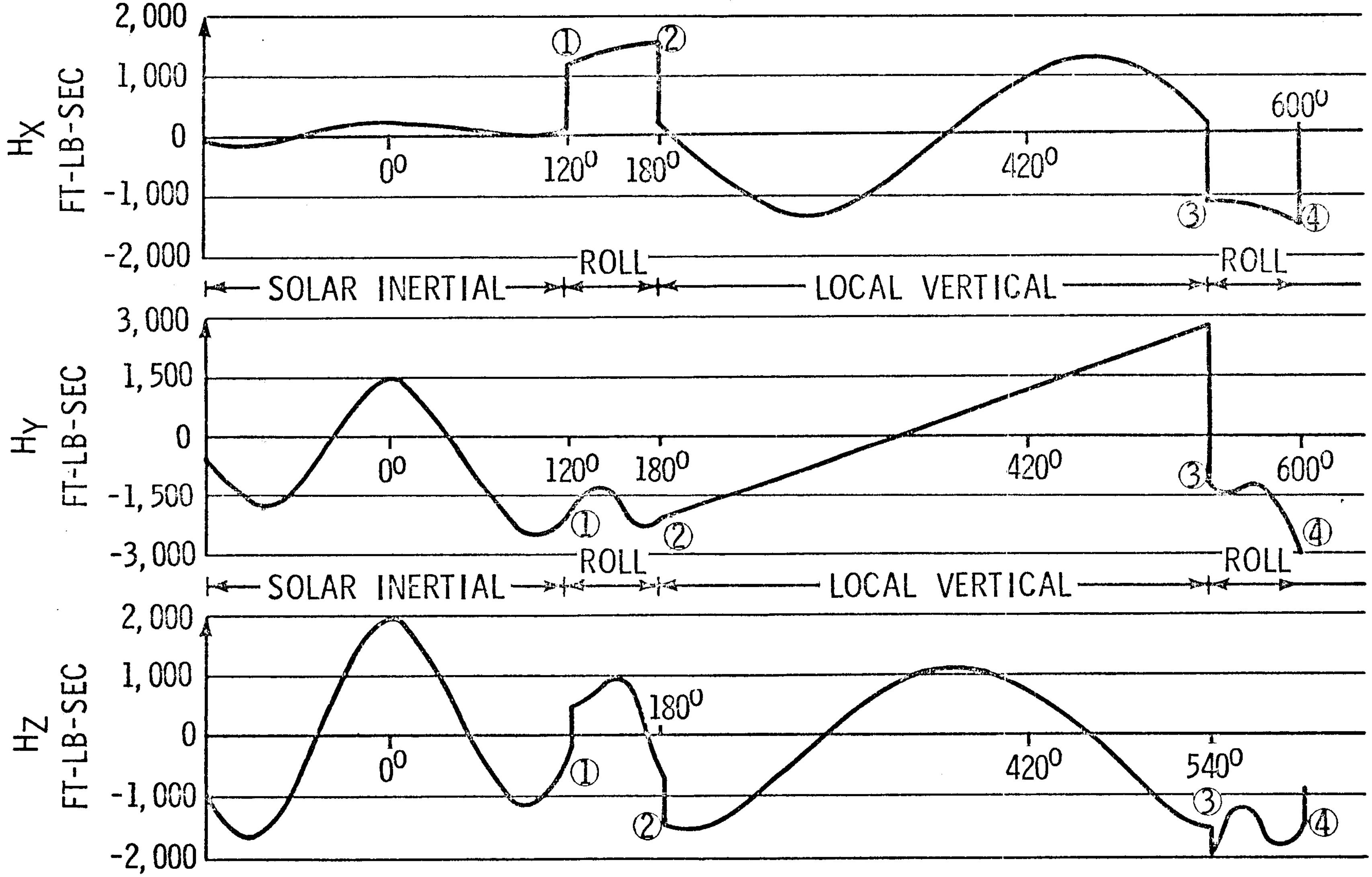
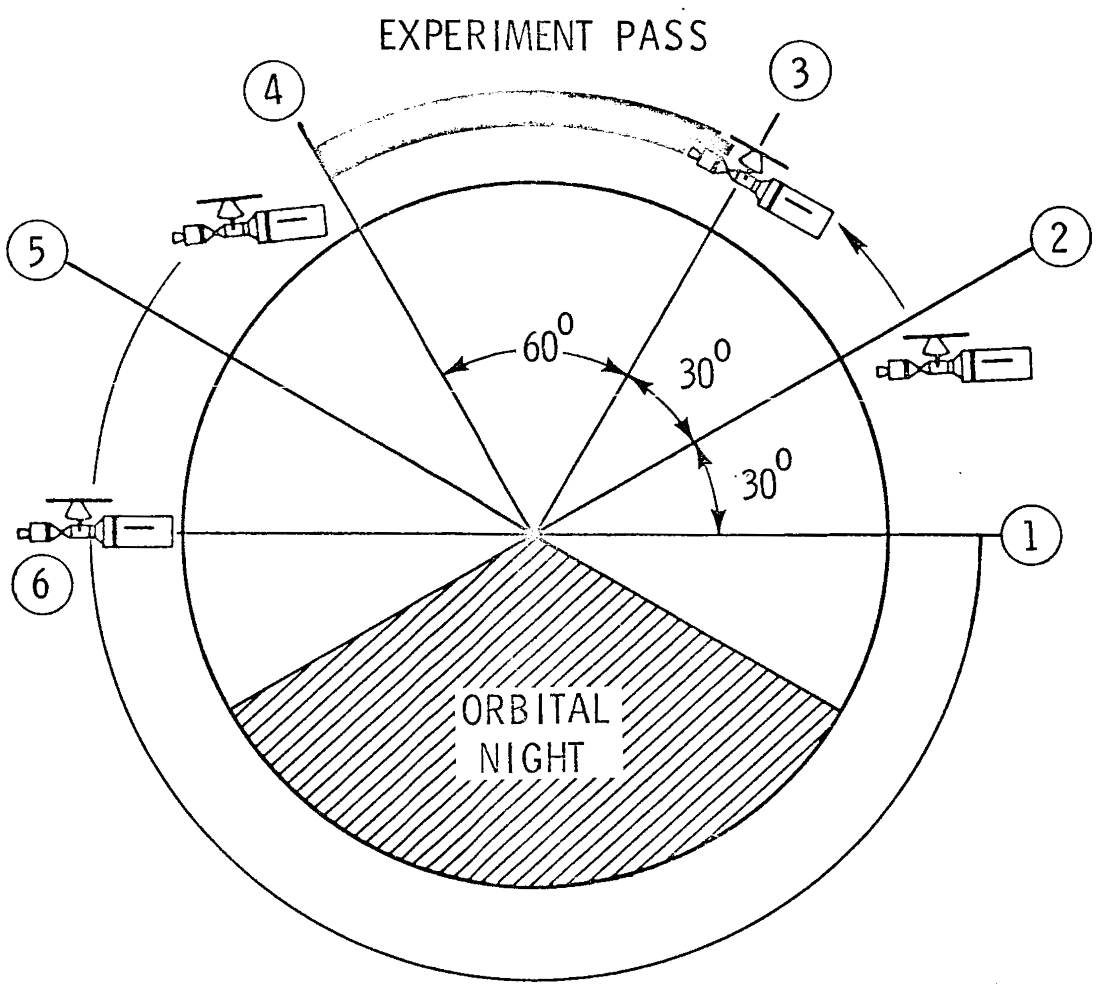



FIGURE 5 - ATTITUDE OPTION 2 - CONTROL OPTION 3
CMG MOMENTUM VARIATION VS. ORBIT ANGLE ($\beta = 45^0$)

ATTITUDE OPTION 3




(1) INITIATE ROLL MANEUVER TO REMOVE $\beta$.

(2) REMOVE ROLL RATE, INITIATE PITCH MANEUVER.

(3) REMOVE PITCH MANEUVER RATE, INITIATE PITCH ORBITAL RATE.

(4) REMOVE PITCH ORBITAL RATE, INITIATE RETURN PITCH MANEUVER RATE.

(5) REMOVE PITCH RATE, INITIATE ROLL RATE.

(6) REMOVE ROLL RATE.

FIGURE 6

# APPENDIX A

## A GRAVITY-GRADIENT DUMP SCHEME

If the local vertical mode is acquired at noon or if successive orbits can be tolerated by the thermal and power systems of the spacecraft, then the dark side of the orbit has no scheduled maneuvers. Further, equation (8) suggests that a Y-axis maneuver might be scheduled such that the momentum which accumulates about this axis can be dumped by the Earth's gravitational field. This is verified by the following example.

Consider the dark side maneuver illustrated in Figure 2. A constant rate is imparted about the spacecraft Y axis at the sunset terminator ② causing the spacecraft axes to move away from the orbit referenced system (ORS). In order that the body axes be realigned with the ORS at the sunrise terminator, the rate is reversed at orbital midnight ③. Then, at the sunrise terminator ④, the return maneuver rate is removed and the spacecraft geometric axes (GS) are realigned with the ORS.

From equation (8), the Y-axis component of the gravity-gradient torque on the spacecraft as it rotates about the Y-axis through an angle θ away from the ORS is given by

$$N_{ggy} = 1.5\omega_o^2[(I_{zz}-I_{xx})\sin 2\theta + 2I_{xz}\cos 2\theta] \tag{A-1}$$

If the sunrise terminator is defined at $t = 2\pi/3\omega_o$ sec and if at this instant the Y axis velocity is changed from $\omega_o$ to $\omega_o(1-\varepsilon)$, then θ is given by

$$\theta = -\varepsilon\omega_o\left(t-\frac{2\pi}{3\omega_o}\right) . \tag{A-2}$$

Now, since the accumulation of bias momentum about this axis is 4645 ft-lb-sec per orbit, there will be approximately 3100 ft-lb-sec accumulated between the terminators. This must be dumped on the dark side and, since the maneuver is symmetrical, half of this amount or 1550 ft-lb-sec must be dumped between the sunset terminator and midnight. From equation (7) the change in Y-axis momentum is equal to the integral of the external torque over time. This leads to the following expression for the required dumping maneuver

$$\text{(A-3)} \quad -1550 = 1.5\omega_o^2 \int_0^{\pi/3\omega_o} [2I_{xz} \cos(2\varepsilon\omega_o\tau) - (I_{zz}-I_{xx}) \sin(2\varepsilon\omega_o\tau)]d\tau,$$

where the substitution $\tau = (t-2\pi/3\omega_o)$ has been made. The solution of (A-3) can be expressed in the form

$$\text{(A-4)} \quad 1535\omega = 0.75\omega_o A \,[\cos\delta - \cos(2\pi\varepsilon/3-\delta)]$$

where:

$$A = [4I_{xz}^2 + (I_{zz}-I_{xx})^2]^{1/2}, \text{and}$$

$$\delta = \tan^{-1}\left(\frac{2I_{xz}}{I_{zz}-I_{xx}}\right) .$$

This is a transcendental equation and must be solved approximately. Using the mass properties of the SVWS leads to the solution (obtained graphically) $\varepsilon = 0.45$. This implies a maximum angular variation from the ORS of -27 degrees at orbital midnight. That is,

$$\text{(A-5)} \quad \theta(\pi/\omega_o) = -27 \text{ degrees.}$$